Lesson 1. Introduction

SA305 – Linear Programming

Spring 2021

What is operations research?

What is operations research?

- "The most influential academic discipline field you've never heard of"

  Boston Globe, 2004

- **Operations Research (OR)** is the discipline of applying advanced mathematical methods to help make better decisions

- "The Science of Better" INFORMS slogan

- "A liberal education in a technological world"

  Thomas Magnanti, former Dean of Engineering at MIT

What is operations research?

- Numerous applications, e.g.
  - logistics
  - manufacturing
  - workforce scheduling
  - finance
  - marketing

OR and the military

- The military uses OR to improve decision making in a variety of ways, e.g.
  - force composition
  - weapon selection
  - search and detection
  - flight operations scheduling
  - training and personnel assignment
- Assessment Division (OPNAV N81) at the Pentagon
- The Naval Postgraduate School has one of the oldest and most well-respected OR departments in the US
- *Naval Research Logistics* is a prominent academic journal featuring research in OR

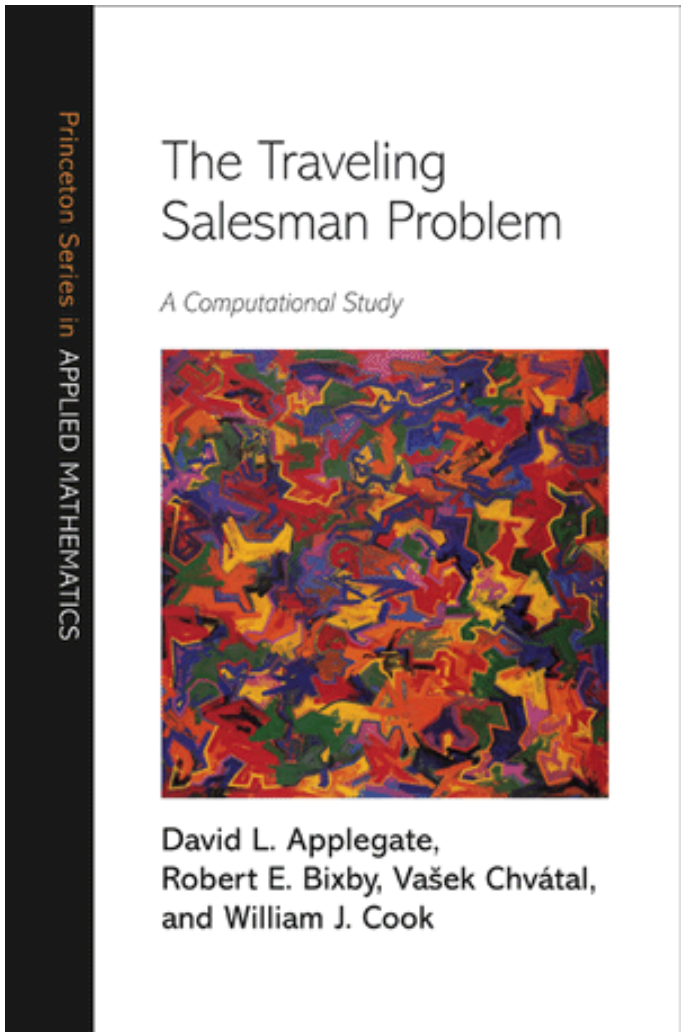
The traveling salesperson problem

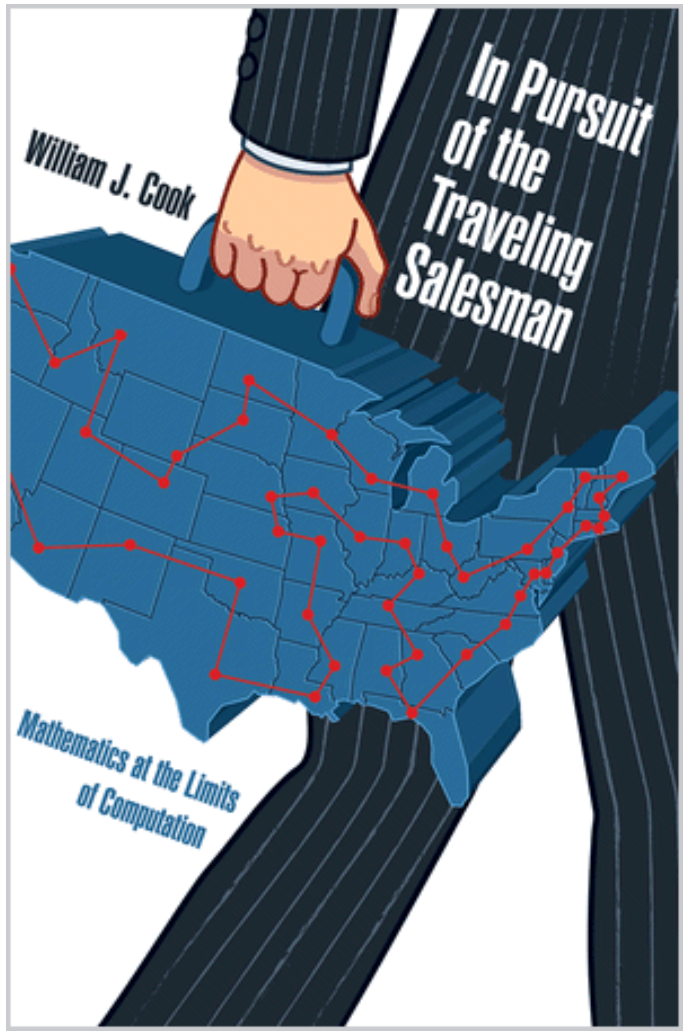
The traveling salesperson problem

- A salesperson located in Annapolis wants to visit clients in each of the 48 state capitals of the continental US and Washington DC
- What is shortest way of visiting all the capitals and then returning to Annapolis?

The traveling salesperson problem

- Entire books have been written on the TSP





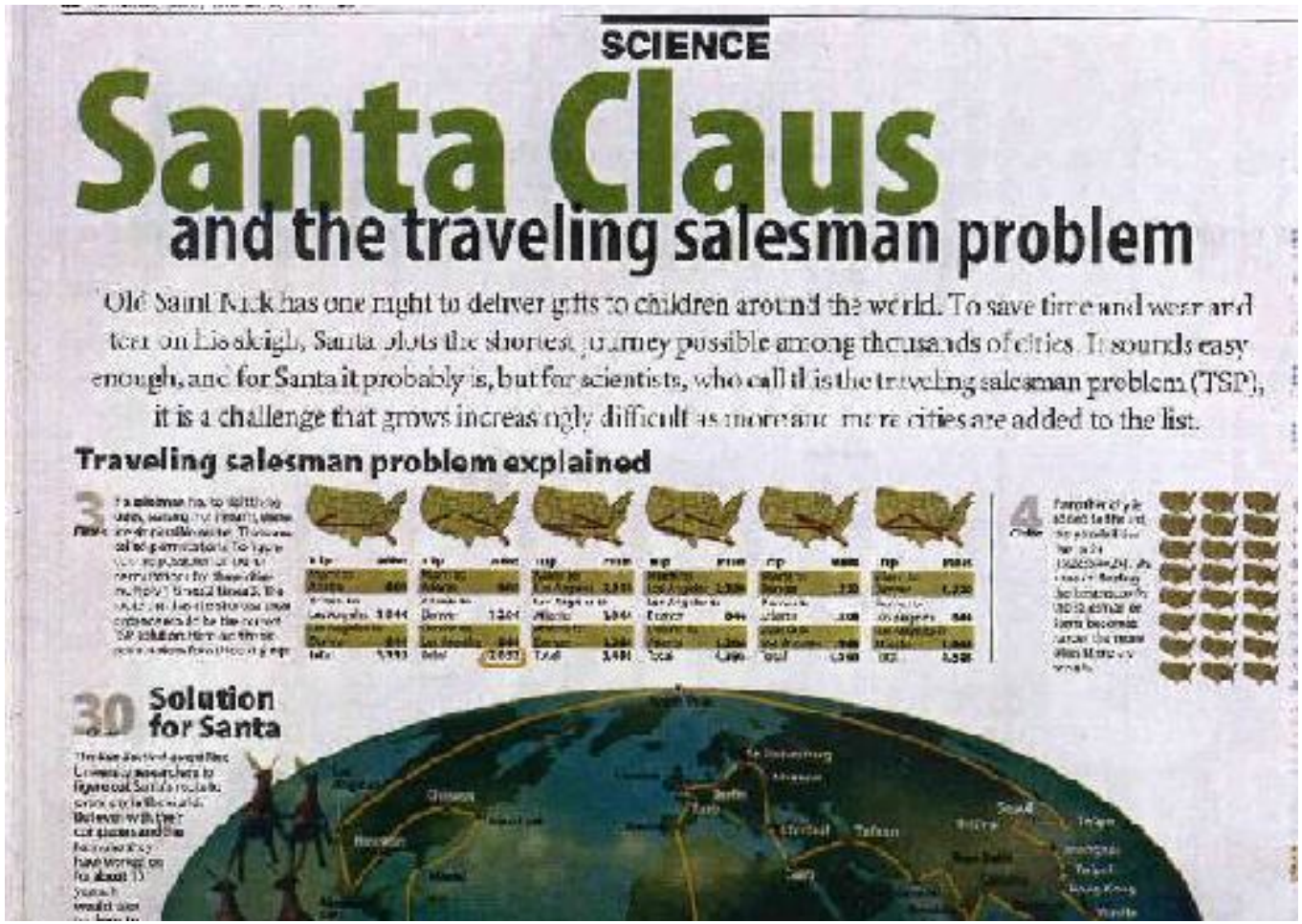
The traveling salesperson problem

- 1962: contest by Proctor and Gamble - best TSP tour through 33 US cities

http://www.math.uwaterloo.ca/tsp

The traveling salesperson problem

- 1998: The Florida Sun-Sentinel's Science page ponders Santa Claus's traveling problem



SCIENCE

Santa Claus
and the traveling salesman problem

Old Saint Nick has one night to deliver gifts to children around the world. To save time and wear and tear on his sleigh, Santa plots the shortest journey possible among thousands of cities. It sounds easy enough, and for Santa it probably is, but for scientists, who call it the traveling salesman problem (TSP), it is a challenge that grows increasingly difficult as more and more cities are added to the list.

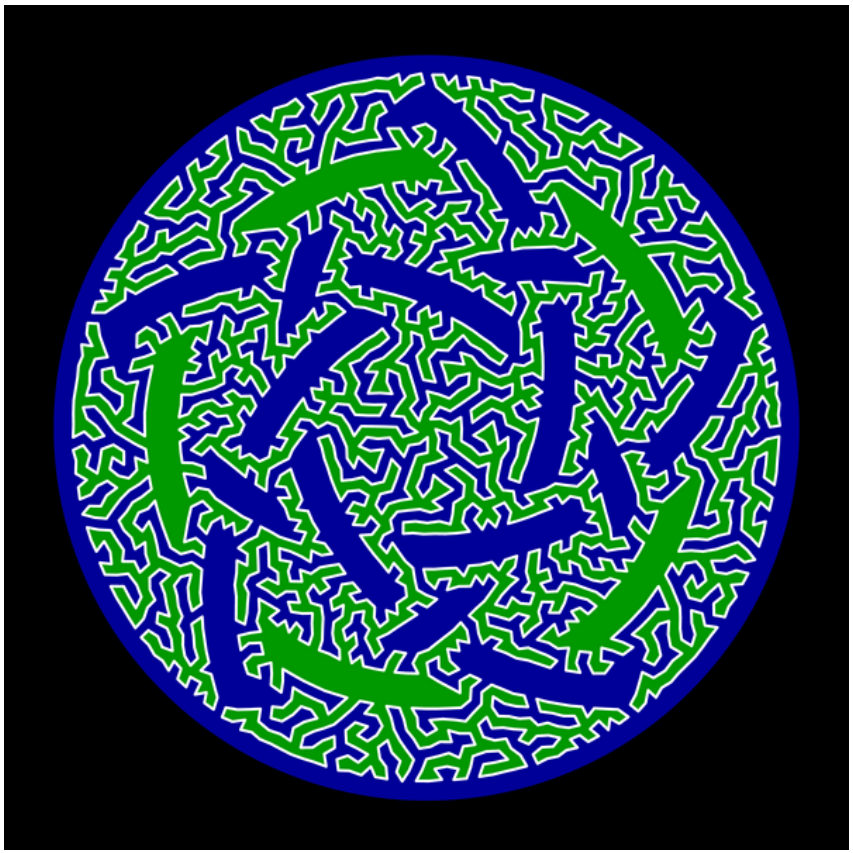
Traveling salesman problem explained

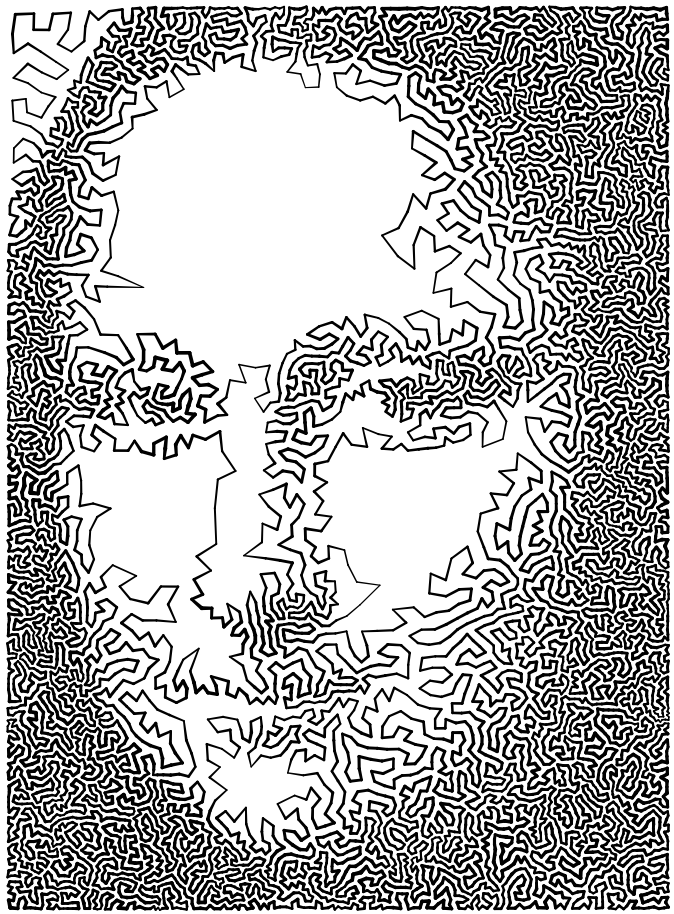
30 Solution for Santa

http://www.math.uwaterloo.ca/tsp

The traveling salesperson problem

- The TSP has even been used to make art





http://www.oberlin.edu/math/faculty/bosch/

http://www.cgl.uwaterloo.ca/~csk/projects/tsp/

The traveling salesperson problem

- One of the most popular problems in operations research
- Numerous applications in expected and unexpected places
  - Circuit board manufacturing
  - Genome sequencing

The traveling salesperson problem

- Your turn! Try to find the shortest way of visiting all the capitals and then returning to Annapolis

The traveling salesperson problem

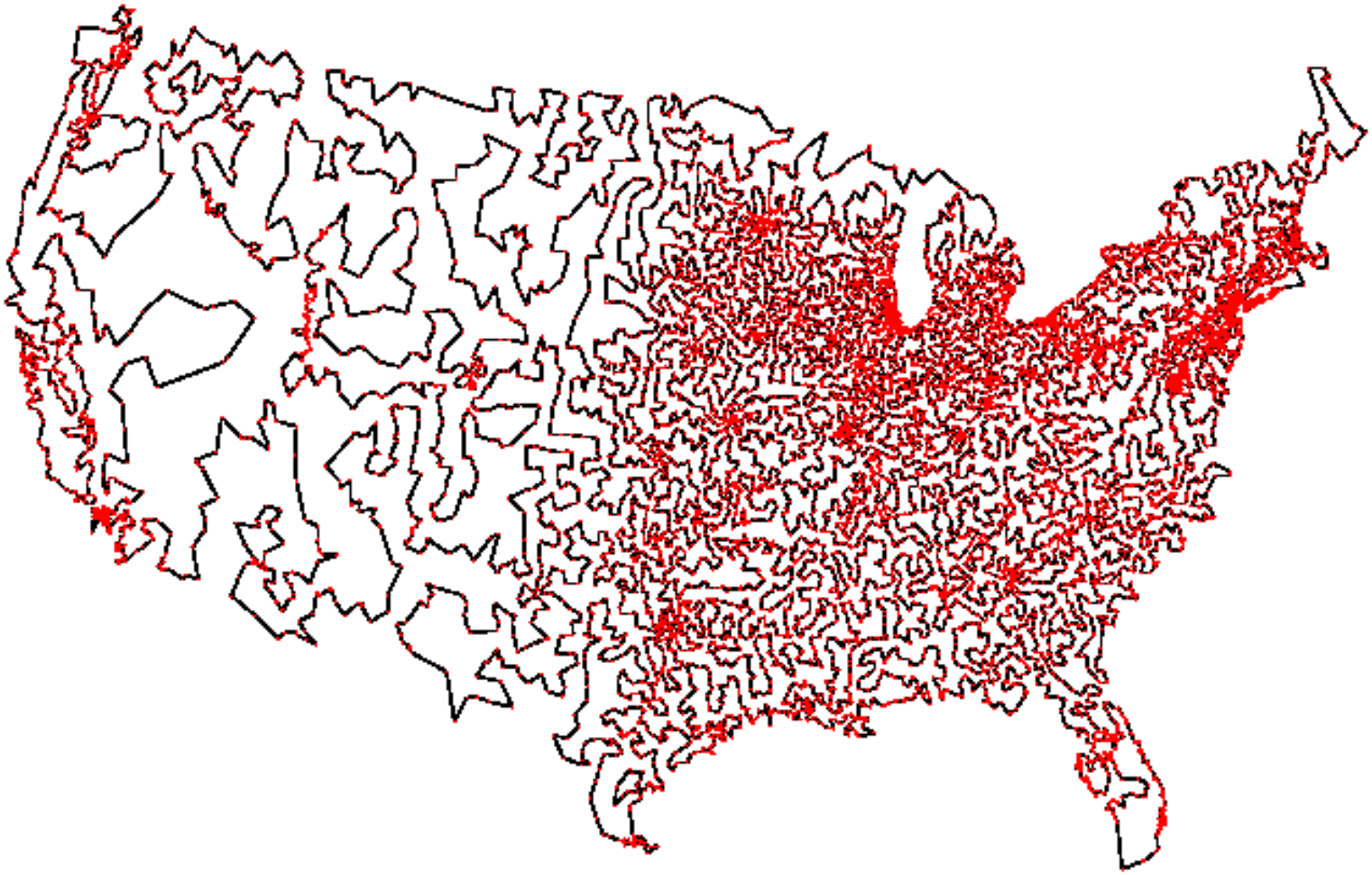
- The solution:

The traveling salesperson problem

- What about 13,509 cities in the US?

The traveling salesperson problem

- What about 13,509 cities in the US?



- Sophisticated mathematical techniques are our best bet

The OR approach

Interpreting

Problem statement and data

Solution

What is the shortest way of visiting all the capitals and then returning to Annapolis?

Modeling

Solving

$$\begin{aligned} \min \quad & \sum_{\{i,j\}\in E} c_{\{i,j\}} x_{\{i,j\}} \\ \text{s.t.} \quad & \sum_{\{i,k\}\in E} x_{\{i,k\}} = 2 \quad \forall i \in N \\ & \sum_{\{i,j\}\in \delta(S)} x_{\{i,j\}} \geq 2 \; \forall S \subseteq N \\ & x_{\{i,j\}} \in \{0,1\} \qquad \forall \{i,j\} \in E \end{aligned}$$

Mathematical model

Goals for this course

- Modeling
  - Recognize opportunities for mathematical optimization
  - Formulate optimization models – linear programs – that capture the essence of the problem
  - Illustrate applications of real-world problems
- Solving
  - Algorithms to solve these mathematical models
- Detailed topic list and schedule is on the syllabus

Optimization is everywhere

- "Minimize" time it takes to get from class to class
- "Maximize" the company's profits
- (Moneyball) "Best" lineup for the Oakland A's
- We are always trying to make **decisions** in a way that meets some **objective** subject to some **constraints**
- Some success stories of optimization helping solve complex real-world decision-making problems ...

Package delivery

- UPS has an air network consisting of 7 hubs, nearly 100 additional airports in the US, 160 aircraft of nine different types
- Decision: aircraft routes, package assignments, flight schedules, etc.
- Objective: minimize delivery times/costs
  maximize delivery throughput
- Constraints: aircraft capacity, fuel requirements, traffic considerations, packages delivered correctly
- UPS credits optimization-based planning tools with identifying operational changes that have saved over $87 million to date, reduced planning times, peak and non-peak costs, fleet requirements

Sports scheduling

- ACC Basketball earns millions in revenue annually, almost all from TV and radio
- TV networks need a steady stream of "high quality" games, NCAA rules, school preferences and traditions
- Decision: who plays who, when, where
- Objective: minimize travel times for teams
  maximize viewership
- Constraints: NCAA rules, travel budgets, school traditions
- Optimization approaches yields reasonable schedules very quickly

Radiation therapy

- High doses of radiation can kill cancer cells and/or prevent them from growing and dividing
- Can also kill healthy cells!
- Radiation can be delivered at different angles and intensities
- Decision: angles + intensities of radiation
- Objective: maximize dose to cancer cells
- Constraints: limit on dose to healthy cells
- Many successes reported using different types of optimization models

Next time...

- Formulating a small optimization model